COMMUNICATIONS TO THE EDITOR

On the Correlation of Data in Nucleate Pool Boiling from a Horizontal Surface

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Depending on the characteristic length of the system, there appear to be two scaling methods used in the analysis of boiling heat transfer. Some investigators in their proposed correlation used the diameter of the pipe or the length of the plate as the characteristic length. This method, however, requires knowledge of the bubble population, which is not yet available analytically. Inasmuch as in nucleate boiling bubbles act as strong agitators and the heat flux appears to be independent of the geometry of the surface, it is also possible to scale boiling heat transfer by considering the action of a single bubble. In this case there is, however, a choice of three characteristic lengths: the critical radius R_0, the bubble radius R, and the diameter D of a bubble at the instant of departure from the heating surface. The purpose of this note is to consider the difference between the latter two.

There is by now enough experimental evidence to indicate that the large heat transfer rates associated with nucleate boiling are a consequence of the microconvection in the superheated liquid sublayer. This motion is caused by the dynamics of bubbles which nucleate and grow in the superheated liquid film. In a theoretical analysis of nucleate boiling it is, therefore, necessary to take into account both processes, nucleation and bubble dynamics, if the similarity to the physical system is to be preserved. It appears that nucleate boiling is a balance between these two processes; at low pressures the motion caused by the growing bubble is more important, but at high pressure the nucleation becomes the controlling factor. In a recent paper (*1*) we have pointed out the physical reason for this. The rate of evaporation ($\rho_v R^2 \dot{R}$) for a given amount of superheat is much higher at low pressures than at high pressures, and the rate of nucleation which depends on the surface tension and the slope of the saturation line increases with an increase of pressure.

Consider now the diameter of a bubble at departure given by the Laplace relation:

$$D = C\sqrt{\frac{\sigma}{g(\rho_L - \rho_v)}} \qquad (1)$$

where C is a constant. The physical significance of this length is that it is related to the maximum size of a bubble for which a static equilibrium exists between the buoyant and adhesive forces. We have already pointed out that at low pressure the dynamic forces are predominant. One can, therefore, expect that these forces would be important in determining the size of a bubble departing from the heated surface, as has been shown in experiments reported by Rohsenow and Clark (*2*) and Ellion (*3*). This can be also shown analytically. However, as the pressure is increased and the dynamic forces decrease, one would expect that the static forces (buoyant and adhesive) would gradually determine the diameter. Since in the pressure range of interest ($0.01P_{crit}$ to $0.8P_{crit}$) this diameter, given by Equation 1, decreases by a factor between 2 and 3, it could be expected that the size of departing bubbles at high pressures would appear approximately constant. This fact was actually observed by Clark (4).

An important fact, however, should not be overlooked. While it appears to be true that at high pressure the diameter of the departing bubble is given by Equation (1), such a bubble could not have originated from *one* nucleating center. It is known that the heat transfer rate increases with an increase in pressure up to approximately 0.35 of the critical pressure. Inasmuch as the rate of evaporation constantly decreases while the diameter D remains approximately constant, the only way to account for the increase of the heat flux is by taking into consideration the nucleation. As mentioned previously, a large increase of nucleating centers (for a given degree of superheat) can be expected with an increase in pressure. This fact has been experimentally observed. A bubble of diameter D would then originate by a coalescence of several small bubbles rather than by the growth from a single nucleus. It can be seen, therefore, that if Laplace's relation is used as the characteristic length the dynamic similarity is not preserved as the pressure is increased.

We have shown in our paper mentioned above that it is possible to scale nucleate pool boiling from a horizontal surface by considering the action of a single bubble. Taking the bubble radius R and radial velocity $\mathring{R}$ as the characteristic length and velocity of the flow system (superheated film), we expressed the Nusselt modulus in its standard form, that is, as a function of the Reynolds and Prandtl moduli:

$$Nu = 0.0015\, Re^{0.62} Pr^{0.33} \tag{2}$$

This method made possible not only the formulation of the Reynolds number in terms of the thermodynamic properties of the vapor and liquid

$$Re = \left[\frac{\Delta T C \rho_L \sqrt{\pi a}}{L \rho_v}\right]^2 \frac{\rho_L}{\mu} \tag{3}$$

but also provided a way to take nucleation into account. We have only to remember that nucleation is a function of the surface energy (σA). The characteristic length in our analysis was the bubble radius R given by

$$R = \frac{\Delta T C \rho_L \sqrt{\pi a}}{L \rho_v} \sqrt{\frac{2\sigma}{\Delta P}} \sqrt[4]{\frac{\rho_L}{\Delta P}} \tag{4}$$

As

$$R\mathring{R} = \left[\frac{\Delta T C \rho_L \sqrt{\pi a}}{L \rho_v}\right]^2 \tag{5}$$

and

$$R_0 = \frac{2\sigma}{\Delta P} \tag{6}$$

then Equation (4) can be put in the following form:

$$R = R_0 \sqrt[4]{\frac{\rho_L R^2 \mathring{R}^2}{\sigma R_0}} \qquad (7)$$

Multiplying and dividing the expression under the root by a length results in a numerator proportional to the kinetic energy and a denominator proportional to the surface energy; the ratio R/R_0 is thus a function of a modified Weber number. It could have been expected that Weber's modulus would also appear in nucleate boiling inasmuch as it always appears when bubbles are considered. As the dynamic forces are important at low pressure, the ratio R/R_0 is large; at high pressure this ratio decreases to unity, which is again in accordance with the previous considerations.

It can be concluded that if in an analysis of nucleate pool boiling from a horizontal surface the bubble radius R and the radial velocity $\mathring{R}$ are taken as the characteristic length and velocity, the two bubble effects, nucleation and liquid agitation, can be taken into account. It is thus possible to preserve the dynamic similarity and correlate the peak nucleate heat transfer rates with a single equation [Equation (2)].

NOTATION

a = thermal diffusivity
c_p = specific heat at constant pressure
D = bubble diameter
g = acceleration due to gravity
k = thermal conductivity
L = latent heat of vaporization
Nu = Nusselt modulus
P = pressure
$\Delta P = P_v - P_\infty$ = pressure difference corresponding to the superheat temperature
Pr = Prandtl modulus
$r = R/R_0$ = dimensionless bubble radius
R = bubble radius
R_0 = radius of critical bubble
$\mathring{R}$ = bubble radial velocity
Re = Reynolds modulus
T_0 = initial superheat temperature
T_∞ = saturation temperature corresponding to pressure on system P_∞
$\Delta T = T_0 - T_\infty$ = superheat temperature
μ = viscosity
ρ = mass density
σ = surface tension

Subscripts

L = liquid
v = vapor

LITERATURE CITED

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